Chance and Chaos
Examples sheet 2
To be handed in Monday May 24

Random matrices. Diagonalise $N \times N$ symmetric, real-valued random matrices $\boldsymbol{H}$. Take $H_{ij} = H_{ji}$ to be Gaussian-distribued random variables with zero mean and variance $\langle H_{ij} H_{kl} \rangle = N^{-1}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\,.$

(a) Calculate the density of states

$$d(E) = \Big\langle \frac{1}{N} \sum_{\alpha=1}^{N} \delta(E - E_\alpha) \Big\rangle$$

where E_α ($\alpha = 1, \ldots, N$) are the eigenvalues of $\boldsymbol{H}$, and $\langle \cdots \rangle$ is an average over realisations of random matrices. Check that the average spacing between neighbouring levels in the vicinity of $E = 0$ is given by $\Delta = 1/[Nd(0)]$.

(b) Calculate the distribution of nearest-neighbour level spacings $s_\alpha = |E_{\alpha+1} - E_\alpha|/\Delta$. Take pairs of levels in the vicinity of $E = 0$.

(c) Calculate the so-called number variance of the spectrum. It is defined as follows. Unfold the spectrum in the vicinity of $E = 0$ by dividing the levels by Δ. After unfolding, the number $n(L)$ of levels in an energy interval (around $E = 0$) of length L will be on average L. Calculate the variance

$$\Sigma_2(L) = \langle n^2(L) \rangle - L^2\,.$$