Dynamical Systems
Examples sheet 4

1. Maximal Liapunov exponent I. An analytical estimate for the maximal Liapunov exponent for the standard map can be obtained as follows. Linearise the standard map around a general point (not necessarily a fixed point), obtaining a matrix of derivatives. Find an approximate expression Λ^+ for the eigenvalue of largest modulus of this matrix, valid for large K. Average $\log(|\Lambda^+|)$ over phase space. Compare your result to the one from the last question of the previous assignment.

2. Maximal Liapunov exponent II. Determine the maximal Liapunov exponent for the Henon map (see assignment no. 2), for $A = 1.4$ and $0 \leq B \leq 0.3$.

3. Hyperbolic dynamics. Consider two-dimensional linear maps on the torus (mod 1) of the form $\boldsymbol{x}_{n+1} = \boldsymbol{M}\boldsymbol{x}_n$, with

$$\boldsymbol{M} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1/2 & 1/2 \\ -1 & 1 \end{pmatrix}, \text{ and } \begin{pmatrix} 3 & 2 \\ 5/2 & 2 \end{pmatrix}.$$

For the three cases, describe the dynamics and determine whether or not it is hyperbolic.

4. Integrable billiards. Explain why each of the three billiard systems shown on p. 165 of the lecture notes is integrable. Explain why the billiards on p. 167 are not.