# Dynamical Systems
# Examples sheet 3

**1. The standard map** was defined in the lectures

$$\begin{aligned} \theta_{n+1} &= \theta_n + p_n \\ p_{n+1} &= p_n + k \sin \theta_{n+1} \end{aligned}$$

where $0 \le \theta_m \le 2\pi$ and $-\pi \le p_m \le \pi$. Note that $\theta_m$ and $p_m$ are defined modulo $2\pi$: whenver $p_{n+1}$ is found to lie outside the interval $[-\pi, \pi]$ suitable multiples of $2\pi$ are added (or subtracted).

(i) Find all $p$-cycles up to $p = 9$ for the standard map for $k = 6$. This must be done numerically, preferably using a Newton search. A suitable algorithm is found in chapter 17 of `http://chaosbook.org`. Plot the number $N_p$ of $p$-cycles you found as a function of $p$ and extract the rate of exponential growth.

(ii) Plot the iterates of all $p$-cycles up to $p = 8$ for $k = 6$ and characterise the distribution of points thus obtained

**2. Invariant manifolds**. On pp. 109, 110 of the lecture notes (outside my office) four properties of invariant manifolds are stated. Prove these statements (i.e. explain why they hold).

Take one or two of the shortest hyperbolic $p$-cycles of the standard map for $k = 6$, determine and plot their stable and unstable manifolds. This must be done numerically, as explained in the lectures. Ask Johan Stäring for advice.

Addendum for those interested: can you find a homoclinic tangency, i.e., a point where $W^u$ and $W^s$ intersect tangentially? Look for near tangencies.

**3. Maximal Liapunov exponent**. Determine the maximal Liapunov exponent for the standard map numerically for $k$ between 2 and 50. Plot it as a function of $k$.