Dynamical Systems
Examples sheet 2

1. Cantor set. What is the box-counting dimension of the Cantor set obtained by removing the middle interval of length 1/2 (instead of 1/3 as in the lecture) of the intervals on the previous stage of the construction?

2. Box-counting dimension I. What are the box-counting dimensions of the sets drawn on p. 75 and p. 76 of the lecture notes?

3. Box-counting dimension II. What is the box-counting dimension of the invariant set in [0.1] for the one-dimensional map given by

$$x_{n+1} = \begin{cases} 4x_n & \text{for } -\infty < x_n \leq 1/2 \\ 2(x_n - 1/2) & \text{for } 1/2 < x_n < \infty \end{cases} ? \tag{1}$$

4. Generalised baker's map I. Write a computer program to take iterates of the generalised baker's map. Choose $\lambda_a = \lambda_b = 1/3$ and $\alpha = 0.4$. For the initial condition$(x_0, y_0) = (1/\sqrt{2}, 1/\sqrt{2})$ iterate the map 100 times and then plot the next 1000 iterates to get a picture of the attractor.

5. Generalised baker's map II. Determine the dimension spectrum D_q for the generalised baker's map (parameters as above).

6. Henon attractor. Consider the two-dimensional Henon map

$$\begin{aligned} x_{n+1} &= A - x_n^2 + By_n \\ y_{n+1} &= x_n \end{aligned}$$

for $A = 1.4$ and $B = 0.3$. Use a computer to generate the attractor. Determine its box-counting dimension D_0.