

Dynamical Systems

Examples sheet 1

1. Cubic map. Write a computer program to make a bifurcation diagram for the cubic map $f(x) = rx(1 - x^2)$ with $r > 0$ (refer to p. 44 of the lecture notes). Plot the bifurcation diagram using as your initial condition $x_0 = 1/\sqrt{3}$. Repeat using $x_0 = -1/\sqrt{3}$. Comment on the result.

2. Natural invariant density I. Determine the natural invariant densities for the tent map, the Bernoulli shift and for the logistic map ($r = 4$) by computer simulation.

3. Natural invariant density II. For the logistic map for $r = 4$ show that the natural invariant density satisfies the equation

$$\rho(x) = \int dx' \delta[x - f(x')] \rho(x').$$

Thus the natural invariant density is an eigenvector of the Perron-Frobenius equation (see lecture notes).

4. Tangent bifurcation. Consider the map $f(x) = r - x^2$. Show that it has a (forward) tangent bifurcation at some value $r = r_0$ at which a stable and an unstable fixed point are created. Find r_0 and the locations of the stable and unstable fixed points. Find r_1 , the value of r at which the stable fixed point created at r_0 becomes unstable.